LETTERS TO THE EDITOR.

[*The Editor does not hold himself responsible for opinions expressed by his correspondents. Neither can he undertake to return, or to correspond with the writers of, rejected manuscripts intended for this or any other part of* NATURE. *No notice is taken of anonymous communications.*]

The Doppler Effect and Carnot's Principle.

IN my letter of March 19 I endeavoured to show that the latent heat absorbed in the production of unit volume of stationary vibration of a particular frequency in a mixed beam of radiation, is not equal to $4q/c$ (where q is the energy stream per sq. cm. per sec., and c the velocity), as would naturally be supposed, but, in consequence of the Doppler effect at the moving mirror or piston, takes the form $T(dp/dT)$, as required by Carnot's principle, where p is the pressure, or the mechanical work per unit volume, and is equal to $2q/c$ for a directly reflected beam under equilibrium conditions. The latent heat, $T(dp/dT)$, may be represented as the sum $(u+p)$ of the intrinsic energy or internal latent heat u and the external work p. As a matter of interpretation, u was identified in my letter with some form of stationary vibration which continued to exist in the medium at the frequency at which it was emitted. Further analysis shows that this is not the case, but that the energy left in the medium conforms exactly to the distribution required by the theory of exchanges. The energy density is $2q/c$ in a directly reflected beam, and is equal to the pressure, but differs from the energy absorbed on emission or evolved on condensation, namely, the latent heat, $T(dp/dT)$, which is the quantity measured experimentally, as previously explained (*Phil. Mag.*, October, 1913, p. 787).

Similarly in the case of full radiation where the energy stream is Q per sq. cm. per sec. in all directions, the energy density is $4Q/c$, and the pressure $4Q/3c$ *for each frequency*, but the latent heat per unit volume is still $T(dp/dT)$ (in place of $16Q/3c$) on account of the Doppler effect, and the energy stream *as measured experimentally* is not Q but $\frac{1}{4}T(dQ/dT)$. With a slight change of viewpoint the consideration of the Doppler effect leads to the exact formulæ and numerical relations already detailed in my previous note (*loc. cit.*), which are now seen to be no longer in conflict with the electromagnetic theory as was at first supposed.

H. L. CALLENDAR.

Imperial College of Science, South Kensington,
March 30.

Lead and the Final Product of Thorium.

IT is now practically certain that the final product of the uranium family of radio-elements is isotopic, or chemically identical, with lead. The constancy of the ratio between lead and uranium, Pb/U, in the case of primary rock-forming minerals of the same geological age, and its sympathetic variation in the case of minerals of different ages, go far to establish this important conclusion. The recent discovery that all the final products of radio-active disintegration fall into Group iv. B of the periodic classification has naturally led to the further suggestion that each one is isotopic with lead.

If lead, or one of its isotopic equivalents, is the final product of the thorium series, then the estimates of geological time hitherto based on the lead-uranium ratio stand in need of a ruthless revision.

Fortunately this does not appear to be necessary, for mineralogical evidence clearly indicates that the presence or absence of thorium in a uranium-bearing mineral does not affect the lead content, which can generally be adequately accounted for by the uranium alone.

It is easy to calculate the relative rates at which uranium and thorium generate their final products, and assuming that the latter are isotopic, to express a given amount of thorium in terms of uranium, and so to arrive at a "total equivalent quantity of uranium," U_e, which also takes thorium into consideration. If then lead is, chemically speaking, the final product of thorium as well as that of uranium, the ratio Pb/U_e ought to be constant for minerals of the same age, and ought to vary in sympathy with the ages if these should differ. I have examined a large number of analyses of radio-active minerals from this point of view, and neither of the above criteria is found to hold. In many cases a large percentage of thorium may be present, but unless uranium is also present, lead is nearly always absent. In the few examples where lead and thorium occur alone, the ratio Pb/Th is variable and bears no relation to the geological age of the minerals.

However, more fully to demonstrate the bearing of evidence of this kind on the problem, Mr. R. W. Lawson, of the Radium Institute, Vienna, and myself are at present estimating the thorium content of a series of Norwegian minerals of Devonian age which had already been analysed for lead and uranium (Proc. Roy. Soc., A, vol. lxxxv., p. 248, 1911). Other workers are busily engaged on determinations of the atomic weights of the lead from uranium and thorium bearing minerals respectively, and there is therefore some likelihood in the near future of a final settlement of the question whether lead is an end-product of thorium or not.

ARTHUR HOLMES.

Geological Department, Imperial College of
Science, London, March 18.

Thermions and the Origin of Solar and Terrestrial Magnetism.

PREVALENT opinion seems to favour decidedly the hypothesis that the chief part of the magnetism of the earth or sun is due to the rotation of all, or a considerable portion of, the matter of which it is constituted. Theories of the magnetisation of matter by rotation fall into two classes: one (*a*) assuming that the substance is magnetic but not necessarily charged, the other (*b*) assuming that the substance is charged but not necessarily magnetic.

(*a*) If the matter is magnetic, consisting of molecular systems with individual magnetic moments differing from zero, rotation about a given axis will, on the electron theory, produce a torque on each individual system, causing it to contribute a magnetic moment parallel to the axis of rotation, and thus magnetising the whole body, if originally neutral, along this axis.

(*b*) Gravitation, or electrical forces, acting differentially on the positive and negative constituents of the matter, or differential centrifugal action, or some other cause, may give rise to a volume-density of electrification throughout the mass of a rotating body, in which case magnetisation, or at least a magnetic field, must result from the convection currents thus formed.

The particular kinds of differential action just mentioned have been proposed before, but I have seen no reference to the fact that an essentially steady electric volume-density must long ago have been produced by the emission of negative electrons from the heated matter of which the earth and sun are composed, and the resulting internal electric field. As the emission increases with the temperature, which increases from the surface inward, it is clear that the volume-density must be of the proper size to account for the polarity of solar and terrestrial magnetism.

© 1914 Nature Publishing Group

It seems probable that both classes of effects are involved in the actual magnetisation in question, though experiment has shown that any effect of class (*a*) is at least exceedingly minute unless the magnetic behaviour of the interior parts of the earth and sun is quite different from that of matter at ordinary temperatures on the surface of the earth.

S. J. Barnett.

The Ohio State University, Columbus,
Ohio, U.S.A., March 12.

A Triangle that gives the Area and Circumference of any Circle, and the Diameter of a Circle equal in Area to any given Square.

It is not possible to measure *exactly* the interminable fraction required for the line BZ in the following figure, but it is quite easy to draw it so nearly that the error is practically immensurable.

First Method.—Draw a line AB = 44, and make BZ = 23. $\frac{23}{44} = 0{\cdot}52272\dot{7}\dot{2}$, which is a little too long.

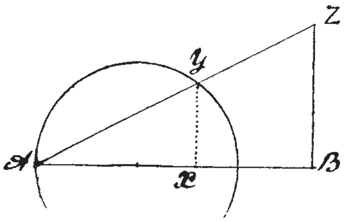
AY will then be short, about 1 in 600,000.
AX will also be short, about 1 in 300,000.

Second Method (with the familiar ratio $\frac{355}{113}$). Make a circle with diameter AB = 11·3, and let AX = $8\frac{7}{8}$. Draw a perpendicular from X to cut the circle in Y. Join AY, and continue the line to Z.

Then the error in AX, the $\frac{1}{4}$ circumference, will be less than 1 in 11,780,000 *in excess*.

The true angle for the line AZ lies between the lines found by the two methods, but the difference is too small for measurement, and in any accurate drawing the lines will appear to coincide.

AX being found, equal to an arc of 90°, a line for any other arc may be found; and the triangle once drawn on a sufficiently large scale, is true for all circles.



Let AB = 1, and at right angles BZ = 0·5227232008 +, join AZ } Angle A. 27° 35′ 49·6″ +

Then, any circle with diameter upon AB and one extremity at A, will cut the line AZ (or AZ produced) in a point Y, making AY the side of a square equal in area to the circle.

Also, a line from Y perpendicular to AB will cut the diameter in a point X, making AX equal to $\frac{1}{4}$ circumference of the circle.

Again, any square with base upon AB and a corner at A, will, with its side opposite to A, cut AZ in a point Y, making AY the diameter of an equal circle.

T. M. P. Hughes.

5 The Croft, Tenby.

The following remarks may help to explain Mr. Hughes's constructions:—

Let AY be a chord of a circle of which AD is a diameter, and let $\angle DAY = \theta$. Then if the square on AY equals the area of the circle, $(2r\cos\theta)^2 = \pi r^2$, and therefore $\cos^2\theta = \pi/4$, $\tan^2\theta = (4-\pi)/\pi$, and

$$\tan\theta = 0{\cdot}5227232,$$

very nearly, as stated. Now if we express this approximate value of $\tan\theta$ as an ordinary continued fraction we find the successive convergents,

$$1/1,\ 1/2,\ 11/21,\ 12/23,\ 23/44,\ \text{etc.},$$

where the first of those not written has four digits in numerator and denominator. Hence, as Mr. Hughes has discovered, 23/44 is a very close approximation to the transcendent number $\sqrt{(4-\pi)}/\sqrt{\pi}$. It seems absurd to speak of a mathematical accident, but we do seem to have something of the kind here. Supposing that a/b is a rational approximation to $\sqrt{(4-\pi)}/\sqrt{\pi}$, we should not expect beforehand a solution correct within about $4{\cdot}10^{-6}$ for values of *a*, *b*, each less than 100.

The second construction is obtained by putting, as an approximation,

$$AX : AD = \pi/4 = 355/4{\cdot}113 = 710/8{\cdot}113 = 8\tfrac{7}{8}/11{\cdot}3.$$

It would be easy to make a set-square with its shorter sides in the ratio 23 : 44, and this could be used for the approximate quadrature and rectification of any given circle. It is interesting to see how the same figure solves both problems to the same degree of exactness (practically). I suppose the error in the set-square could be reduced to 0·1 per cent., or less; the question is, what percentage of error is likely to occur in using it. For the rectification we have to draw the perpendicular YX; it seems to me that for the quadrature we are likely to obtain the most accurate results by using a straight edge as well as the set-square; that is to say, we should not try to adjust the set-square without first placing a straight edge along a diameter of the circle. If this is so, the graphical solutions of both problems are likely to be affected by the same percentage of error; because to obtain X, after marking Y, we have only to slide the set-square along the straight edge until a shorter side goes through Y; and if we repeat the manipulation several times, I do not think the error in finding X, regarded as a distance from the true position, can be so much as five times the error in finding Y, or conversely. Of course, by "the same percentage of error" I mean here that the two errors, on the same scale, are of the orders $\pm a{\cdot}10^{-n}$, $\pm b{\cdot}10^{-n}$, where *a*, *b* both lie above 1, while neither of them is equal to, or exceeds, 5.

G. B. Mathews.

New Units in Aerology.

With reference to Prof. McAdie's letter in Nature of March 19, p. 58, I should like to point out that throughout my "Thermodynamics," published in 1878, a megadyne per square centimetre is used as the unit of pressure, and it is termed a c.g.s. atmosphere. Ever since 1888, when the B.A. committee (of which I was a member), adopted the barad, I have employed in my lectures the above pressure unit under the name of megabarad. The corresponding unit of work, and also of heat, adopted in the book is the megalerg. Megerg to my ear is too cacophonous for use.

Robert E. Baynes.

Christ Church, Oxford, March 25.

PROGRESS IN WIRELESS TELEPHONY.

The attention of telephonic engineers has of late years been very closely directed to the improvement of the line wire in ordinary telephony. Apart from the imperfections of the telephone transmitter and receiver, *per se*, a very considerable effect is produced on the transmitted speech by the line itself if it is at all long. This action, from an electrical point of view, consists in the distortion of the wave form of the current

© 1914 Nature Publishing Group